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CORRIGENDUM: THE CAUCHY PROBLEM FOR QUASI-LINEAR PARABOLIC SYSTEMS REVISITED

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ABSTRACT. — In [1, p. 1666], the proof of Theorem 6 had a misprint, which turned into a wrong argument that we correct here.

RÉSUMÉ (Corrigendum : Le problème de Cauchy pour les systèmes paraboliques quasi-linéaires : une nouvelle approche)

Dans [1, p. 1666], la démonstration du théorème 6 comportait une coquille qui a donné lieu à un raisonnement erroné, que nous corrigeons ici.

Let us first recall a few useful notations. We use here the Besov spaces $B_{p,1}^s$ defined in [1, App. B]. For any given $T > 0$ and $p \in [1, \infty)$, the analogue of the solution space E_T^s in the Besov setting is:

$$\mathbb{E}_T^s := \mathcal{C}^0([0, T]; B_{p,1}^s) \cap L^1(0, T; B_{p,1}^{s+2}),$$

while the analogue of the exterior force space Y_T^s is:

$$\mathbb{Y}_T^s := L^1(0, T; B_{p,1}^s).$$

Section 7.4 of [1] intended to define a fixed-point procedure leveraging on [1, Th. 7]. We propose below a complete rewriting of this subsection.

7.4. CONCLUSION. — To conclude the proof we shall use the linear estimate provided by [1, Th. 7] to implement a fixed point argument. Let us set

$$\mathbb{G}_T^p := \mathbb{E}_T^{d/p} \cap \mathcal{C}^0(Q_T).$$

Given $(U^0, F) \in B_{p,1}^{d/p} \cap \mathbb{Y}_T^{d/p}$, we consider the following map

$$\begin{aligned} \Theta : \mathbb{G}_T^{d/p} &\longrightarrow \mathbb{E}_T^{d/p} \\ U &\longmapsto U^*, \end{aligned}$$

where U^* is the only element of $\mathbb{E}_T^{d/p}$ solving $L_{A(U)}U^* = F$ with $U(0) = U^0$. We define as before the reference solution $U_F := \Theta(0)$.

Before starting the fixed point procedure, let us check that Θ maps $\mathbb{G}_T^p$ onto itself. We recall that $U^* := \Theta(U)$ solves

$$\partial_t U^* = \sum_k \partial_k (A(U) \partial_k U^*) + F,$$

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and we know that F belongs to $\mathbb{Y}_T^{d/p}$. Let us prove that the same information holds for $\partial_k(A(U))\partial_k U^*$. We write

$$\partial_k(A(U)\partial_k U^*) = A(U)\partial_k^2 U^* + A'(U)\partial_k U\partial_k U^*,$$

and we know that smooth functions are continuous over $B_{p,1}^{d/p}$, so since $B_{p,1}^{d/p}$ is an algebra. It follows that

$$\|A(U)\partial_k^2 U^*\|_{\mathbb{Y}_T^{d/p}} \lesssim \|A(U)\|_{L_T^\infty(B_{p,1}^{d/p})} \|\partial_k^2 U^*\|_{\mathbb{Y}_T^{d/p}} \lesssim \Phi(\|U\|_{L_T^\infty(B_{p,1}^{d/p})}) \|U^*\|_{\mathbb{Y}_T^{(d/p)+2}}$$

with Φ smooth and increasing. Similarly,

$$\|A'(U)\partial_k U\partial_k U^*\|_{\mathbb{Y}_T^{d/p}} \lesssim \|A'(U)\|_{L_T^\infty(B_{p,1}^{d/p})} \|\partial_k U\|_{L_T^2(B_{p,1}^{(d/p)+1})} \|\partial_k U^*\|_{L_T^2(B_{p,1}^{(d/p)+1})}.$$

We thus find that $\partial_t U^*$ belongs to $L^1(0, T; B_{p,1}^{d/p})$, which implies the expected result since $B_{p,1}^{d/p}$ is embedded in the space of continuous functions. In the following, we restrict our attention to the space $\mathbb{E}_T^{d/p}$, as the estimates can then be extended to the full space $\mathbb{G}_T^p$ by the same argument on the time derivative.

Consider U_1 in the $\mathbb{E}_T^{d/p}$ closed ball of radius r centered at U_F and set $V := U_1^* - U_F$. The same argument as in [1, Cor. 2.5] allows to replace the continuity constant $[A(U_F)]_0$ by $\|U_F\|_{\mathbb{G}_T^p}$, so that

$$\|V\|_{\mathbb{E}_T^{d/p}} \lesssim_{T, \|U_F\|_{\mathbb{G}_T^p}, \omega_{U_F}} \|V(0)\|_{B_{p,1}^{d/p}} + \|L_{A(U_F)} V\|_{\mathbb{E}_T^{d/p}}.$$

Since

$$L_{A(U_F)} V = L_{A(U_1)} V + \sum_k \partial_k (A(U_1) - A(U_F)) \partial_k V,$$

we find, thanks to [1, Prop. B.2],

$$\begin{aligned} \|V\|_{\mathbb{E}_T^{d/p}} &\lesssim_{T, \|U_F\|_{\mathbb{G}_T^p}, \omega_{U_F}} \|L_{A(U_1)} V\|_{\mathbb{E}_T^{d/p}} + \|A(U_1) - A(U_F)\|_{L^\infty(Q_T)} \|V\|_{\mathbb{Y}_T^{(d/p)+2}} \\ &\quad + \int_0^T \| [A(U_1) - A(U_F)](t) \|_{B_{p,1}^{(d/p)+2}} \|V(t)\|_{B_{p,1}^{d/p}} dt. \end{aligned}$$

It follows that, if r is small enough (depending only on T and U_F), then

$$\begin{aligned} \|V\|_{\mathbb{E}_T^{d/p}} &\lesssim_{T, \|U_F\|_{\mathbb{G}_T^p}, \omega_{U_F}} \|L_{A(U_1)} V\|_{\mathbb{E}_T^{d/p}} \\ &\quad + \int_0^T \| [A(U_1) - A(U_F)](t) \|_{B_{p,1}^{(d/p)+2}} \|V(t)\|_{B_{p,1}^{d/p}} dt. \end{aligned}$$

But

$$L_{A(U_1)} V = \sum_k \partial_k (A(U_1) - A(0)) \partial_k U_F,$$

so again, thanks to [1, Prop. B.2],

$$\begin{aligned} \|V\|_{\mathbb{E}_T^{d/p}} &\lesssim_{T, \|U_F\|_{\mathbb{G}_T^p}, \omega_{U_F}} \|A(U_1) - A(0)\|_{L^\infty(Q_T)} \|U_F\|_{\mathbb{Y}_T^{(d/p)+2}} \\ &\quad + \int_0^T \| [A(U_1) - A(0)](t) \|_{B_{p,1}^{(d/p)+1}} \|U_F(t)\|_{B_{p,1}^{(d/p)+1}} dt \\ &\quad + \int_0^T \| [A(U_1) - A(U_F)](t) \|_{B_{p,1}^{(d/p)+2}} \|V(t)\|_{B_{p,1}^{d/p}} dt. \end{aligned}$$

In the right-hand side, the two first lines are small as soon as both T and r are small: for the first one we can split $A(U_1) - A(0)$ into $A(U_1) - A(U_F) + A(U_F) - A(0)$ and use the triangular inequality while for the second one, we use the embedding of $\mathbb{E}_T^{d/p}$ into $L^2([0, T]; B_{p,1}^{d/p+1})$ by interpolation. Then the last line is handled, thanks to a Gronwall estimate, and we can find T small enough so that $\|V\|_{\mathbb{E}_T^{d/p}} \leq r$.

Similarly, we can prove that Θ is Lipschitz on the ball of $\mathbb{E}_T^{d/p}$ centered at U_F and of radius r with a Lipschitz constant smaller than one (for r and T small enough). This follows the same lines as the above computations: we fix U_1 and U_2 such that $\|U_i - U_F\|_{\mathbb{E}_T^{d/p}} \leq r$ and we note that thanks to [1, Prop. B.2]

$$\begin{aligned} \|U_1^* - U_2^*\|_{\mathbb{E}_T^{d/p}} &\lesssim_{T, \|U_1\|_{G_T^p}, \omega_{U_1}} \int_0^T \|(A(U_1) - A(U_2)(t))\|_{B_{p,1}^{(d/p)+1}} \|U_2^*(t)\|_{B_{p,1}^{(d/p)+1}} dt \\ &\quad + \int_0^T \|(A(U_1) - A(U_2)(t))\|_{L^\infty} \|U_2^*(t)\|_{B_{p,1}^{(d/p)+2}} dt \\ &\lesssim_{T, r, \|U_F\|_{G_T^p}, \omega_{U_F}} \|U_1 - U_2\|_{\mathbb{E}_T^{d/p}} (r + \|U_F\|_{L_T^1 B_{p,1}^{(d/p)+2}}) \\ &\quad + \|U_1 - U_2\|_{\mathbb{E}_T^{d/p}} (r + \|U_F\|_{L_T^2 B_{p,1}^{(d/p)+1}}), \end{aligned}$$

where we have used that, since $\|U_1 - U_F\|_{\mathbb{E}_T^{d/p}} \leq r$, then in particular for any (t, x) and (t', x') in Q_T there holds

$$|U_1(t, x) - U_1(t', x')| \leq 2r + |U_F(t, x) - U_F(t', x')|,$$

whence

$$\omega_{U_1} \leq r + \omega_{U_F}.$$

The result follows, choosing r and T small enough. $\square$

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