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Corrigendum: Mirror symmetry for extended affine Weyl groups

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CORRIGENDUM: MIRROR SYMMETRY FOR  
EXTENDED AFFINE WEYL GROUPS

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ABSTRACT. — We correct an error in [1]. There was a typo in the exponent of one of the factors appearing in the expression of the Lyashko–Looijenga degrees in Corollary 5.2; the resulting wrong formula was then used in Table 5.1 to tabulate these degrees for all Dynkin types. We provide here the correct expressions for both Corollary 5.2 and Table 5.1.

RÉSUMÉ (Corrigendum : Symétrie miroir pour les groupes de Weyl affines étendus)

Nous corrigeons une erreur dans [1]. Il y a une coquille dans l'exposant de l'un des facteurs apparaissant dans l'expression des degrés de Lyashko–Looijenga au Corollaire 5.2; la formule erronée qui en résulte est utilisée dans le tableau 5.1 pour lister ces degrés pour tous les types de Dynkin. Nous indiquons ici les expressions correctes pour le corollaire 5.2 et le tableau 5.1.

The formula in the statement of [1, Cor. 5.2] contains a computational error, whereby the exponent of  $(\omega_{\bar{k}}, \omega_{\bar{k}})$  in the numerator should have been  $\ell_{\mathcal{R}} + 1$  (and not  $\ell_{\mathcal{R}}$ ). We give the rectified statement below.

COROLLARY 5.2. — *The degree of the LL-map of the Hurwitz stratum  $M_{\omega}^{\text{LG}}$  is*

$$\frac{(\ell_{\mathcal{R}} + 1)! (\omega_{\bar{k}}, \omega_{\bar{k}})^{\ell_{\mathcal{R}} + 1}}{\prod_{j=1}^{\ell_{\mathcal{R}}} (\omega_j, \omega_{\bar{k}})}.$$

The incorrect formula was used to tabulate the topological degrees for all Dynkin types other than  $A_{\ell}$  in Table 5.1. We correct those values in the table below.

*Acknowledgements.* — We became aware of an issue with the formula as presented in [1, Cor. 5.2] after the appearance of [2]. We are very grateful to A. Takahashi for correspondence related to this.

MATHEMATICAL SUBJECT CLASSIFICATION (2020). — 53D45, 14B07, 20H15.

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TABLE 5.1. Lyashko–Looijenga degrees for all Dynkin types.

$\mathcal{R}$	$g_\omega$	$\mathfrak{n}_\omega$	$d_{g_\omega, \mathfrak{n}_\omega}$	$\iota_\omega(\mathcal{M}_{\mathcal{R}}^{\text{DZ}})$	$\text{deg}(\text{LL})$
$A_\ell$	0	$(\bar{k} - 1, \ell - \bar{k})$	$\ell + 1$	$\mathcal{H}_{g_\omega, \mathfrak{n}_\omega}^{[\mu]}$	$\frac{(\ell + 1 - \bar{k})^{\ell+1-\bar{k}} (\bar{k})^{\bar{k}} \ell!}{(\bar{k} - 1)! (\ell - \bar{k})!}$
$B_\ell$	0	$(\ell - 2, \ell - 2, 1)$	$2\ell + 1$	$(\mathcal{H}_{g_\omega, \mathfrak{n}_\omega}^{[\mu]})^{\mu_2}$	$2(\ell + 1)\ell(\ell - 1)^\ell$
$C_\ell$	0	$(\ell - 1, \ell - 1)$	$2\ell$	$(\mathcal{H}_{g_\omega, \mathfrak{n}_\omega}^{[\mu]})^{\mu_2}$	$(\ell + 1)\ell^{\ell+1}$
$D_\ell$	0	$(\ell - 3, \ell - 3, 1, 1)$	$2\ell + 2$	$(\mathcal{H}_{g_\omega, \mathfrak{n}_\omega}^{[\mu]})^{\mu_2}$	$4\ell(\ell^2 - 1)(\ell - 2)^{\ell-1}$
$E_6$	5	$(5, 5, 2, 2, 2, 2)$	42	(A.1)	$2^4 \cdot 3^7 \cdot 5 \cdot 7$
$E_7$	33	$(11, 5, 3, 11, 5, 3, 1, 1, 3, 3)$	130	(A.2)	$2^{14} \cdot 3^4 \cdot 5 \cdot 7$
$E_8$	128	$(29, 29, 14, 14, 14, 14, 14, 14, 9, 9, 9, 9, 5, 5, 4, 4, 4, 4, 4, 2, 2, 0, 0)$	518	[6, 7]	$2^5 \cdot 3^6 \cdot 5^6 \cdot 7$
$F_4$	4	$(5, 5, 2, 2, 2, 2)$	36	$(3.40);$ $(M_{[100000]_{E_6}}^{\text{LG}})^{\mu_2}$	$2^4 \cdot 3^4 \cdot 5$
$G_2$	0	$(1, 1, 1)$	7	$(3.41);$ $(M_{[1000]_{D_4}}^{\text{LG}})^{S_3}$	$2^3 \cdot 3^2$

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